Passive Control Design for Distributed Process Systems: Theory and Applications

Antonio A. Alonso
Dept. of Chemical Engineering, Universidade de Vigo, 36200 Vigo, Spain

Julio R. Banga and Isidro Sanchez
Chemical Engineering Lab, IIM-CSIC, 36208 Vigo, Spain

A recently developed theory linking passivity with the second law of thermodynamics was used to develop a robust control design methodology for process systems with states distributed in time and space. Asymptotic stabilization of the infinite dimensional state thus can be accomplished for convection−diffusion processes with nonlinear production terms. Two examples representative of these phenomena were considered: chemical reactors and thermal treatments induced by electromagnetic fields. The first case shows how mixing and reactor size are critical control design parameters. If mixing is complete, at least in some direction, exponential stabilization can be achieved by high gain control. If not, stabilization is still possible for reaction domains smaller than a critical volume. In the second case, the electromagnetic field power supplied can always be manipulated to preserve passivity for any domain size. Two important consequences are that the infinite dimensional state can be reconstructed at arbitrary precision by robust observers and that control of the energy inventory will suffice to provide asymptotic stabilization. Theoretical justification of these findings is given on a general framework and illustrated through simulation experiments.

Introduction

The control of distributed process systems (DPS) has received considerable attention over the years from both the theoretical and application points of view [see Ray (1978) and Balas (1982), or the more recent reviews by Lasiecka (1995) and Christofides and Daoutidis (1997a)]. A number of processes in the chemical industry, such as reactors or separation units, can be considered as infinite dimensional systems, this being particularly true when the operation is carried out on short time scales. Interesting case studies can be found in both the food industry and materials technology. Examples of these processes include extrusion and thermal processing by steam (sterilization, pasteurization, etc.), electric fields (ohmic heating), and microwave processing.

The main difficulty associated with the control of DPS derives from the fact that only a finite number of actuators and sensors are available to keep the infinite dimensional state under control. Thus most efforts have been directed at developing efficient state reconstruction techniques and compact system descriptions to be used in control design. Different lumping techniques and observers for parabolic systems were discussed by, among others, Cooper et al. (1986) and Dochain et al. (1992, 1997). Gay and Ray (1995) used results from integral theory to reconstruct kernels of parabolic operators and applied them to control temperature in a metallurgical heating process. Chen and Chang (1992) proposed the use of spectral decomposition techniques to develop reduced-order models for diffusion−convection nonlinear processes by exploiting the time-scale separation property of elliptic operators. Based on that model, a smooth nonlinear feedback was induced to improve the dynamic response of the process. This concept was extended by Christofides and Daoutidis (1996, 1997b) and Christofides (1998) to develop stabilizing finite dimensional nonlinear controllers for distributed systems. Recently, Shvartsman and Kevrekidis (1998) demonstrated the effectiveness of linear control methods to stabilize limit cycles in a diffusion−reaction system.

From a different perspective, Alonso and Ydstie (1996) and Ydstie and Alonso (1997) showed the possibility of robust

Correspondence concerning this article should be addressed to A. A. Alonso.

stabilization of distributed systems with convection, diffusion, and reaction, by means of very simple control structure that do not require detailed information about the system dynamics. The basic ingredient of this theory is the second law of thermodynamics which, by ensuring dissipation, keeps the state (the field) bounded (Ydstie and Alonso, 1997). In addition, convergence to stationary states can be guaranteed, provided the system is passive in the sense of systems theory. Passivity is the main tool used by Sepulchre et al. (1997) in the context of nonlinear finite dimensional systems to develop a constructive approach to control design. It implies the existence of a storage function V, bounded from below and dependent on the state that for each time interval $t \in [t, t + \tau]$ satisfies

$$V(t+\tau) - V(t) \leq \int_t^{t+\tau} \varsigma(t)\, ds, \tag{1}$$

where $\varsigma = y^T w$ is known as the supply, y are the outputs (or measurements), and w the inputs or manipulated variables. Under this condition, control design reduces to finding appropriate mappings $w = \phi(y)$, with ϕ chosen such that $y^T \phi(y) < 0$. Some additional conditions will then ensure that V, and therefore the state, will converge. This is so provided y can be measured and a function ϕ can be constructed. In finite dimensional systems, passivity formalism gained popularity for the simplicity and robustness of the resulting control structures (Sira-Ramirez, 1998). Applications include the control and observation of electromechanical systems (Ortega et al., 1998; Fossen and Strand, 1999) and chemical reactors (Sira-Ramirez, 1998). In the context of chemical plants, passivity was combined with thermodynamics to construct stabilizing mass and energy inventory controllers (Farshman et al., 1998) and to derive general structural stability conditions for separation networks (Hangos et al., 1999).

Unfortunately, the logic just described is far more difficult to apply to the systems we are considering in this article. First, y is now infinitely dimensional and cannot be measured everywhere. Second, w might be finite dimensional, which raises difficulties in the search of appropriate functions ϕ compatible with the control objectives (usually associated with the control of the complete, infinite dimensional, state). In order to make inequality 1 suitable for the analysis of infinite dimensional systems, supply for passive systems will be defined as

$$\varsigma = \langle y, w \rangle_{\mathcal{V}} = \int_{\mathcal{V}} y^T w \, d\mathcal{V}, \tag{2}$$

where $\mathcal{V}$ stands for the volume of the domain. This modification generalizes the standard definition to allow infinite dimensional input–output sets to be included. For nondistributed measurements and actuators, supply reverts to the previous definition. In this article, inequalities of the form of Eq. 1 will be shown to exist for a general class of distributed process systems. Representative examples of this class include tubular reactors and thermal processes where energy is supplied by electric or electromagnetic fields. The storage we employ derives from the second law of thermodynamics as proposed by Ydstie and Alonso (1997). For completeness, its definition and properties are presented in the Appendix. On this basis, passivity and exponential stabilization will be established as a function of a reduced number of design parameters. These include the volume of the domain, and the transfer and production rates of chemical species and energy. The proposed passivity framework is used to design simple observation and control schemes that will ensure exponential convergence of the field to a preselected reference. For distributed inputs, stabilization can be achieved through high gain control. When this is not the case, exponential convergence can still be induced by controlling the inventory of the system.

The article is outlined as follows: process description and assumptions will be discussed in the next section. Next, we present the fundamental result establishing conditions under which the class of systems considered becomes passive. Its consequences for controller design will be described. Finally, these ideas will be illustrated with some representative examples, such as tubular reactors and microwave thermal processing.

Distributed Process Systems: Description and Assumptions

The systems we examine in this article are those that satisfy conservation principles for mass energy and momentum. These properties will be grouped into an n-dimensional vector z that will be referred to as the field. Each element of this vector will be considered a function of time and space obeying equations of the form

$$z_t + \nabla(\boldsymbol{v} z + J) = \sigma(z, u) \tag{3}$$

on a compact 3-dimensional spatial domain $\mathcal{V}$ with smooth boundary $\mathcal{B}$. In Eq. 3, both convective and diffusive fluxes are considered, $\boldsymbol{v}$ being the fluid velocity field and J a vector of dispersive fluxes for energy and chemical species. We assume that momentum is decoupled from the rest of the field so that $\boldsymbol{v}$ is only a function of the position coordinates. The term σ defines the production of heat or particular chemical species being formed or disappearing by chemical reaction. The elements of the production term will be, in general, nonlinear functions of the field itself and possibly of some function $u(p, t)$ (p denoting spatial coordinates) with a physical meaning that will become clear later on. Diffusive fluxes are assumed to obey Onsager-type relationships of the form (deGroot and Mazur, 1962)

$$J = L(z) X \tag{4}$$

$$X = \nabla A = M(z) \nabla z. \tag{5}$$

Matrix $L(z)$ includes diffusion/dispersion coefficients associated with the properties contained in z. In Eq. 4 and 5 X is known in irreversible thermodynamics as the vector of thermodynamic forces [see, for instance, Jou et al., (1993)]. This term is related to the field z through its dual representation A. Physically, A represents the set of intensive properties such as temperature and chemical potentials, associated with the set of densities for energy and mol numbers. Duality implies a one-to-one map between both representations and is justified by the fact that the entropy density of the system $s(z)$ is a strictly concave function of the field (Callen, 1985). From the Gibbs-Duhem relation, it follows that the intensive properties are the partial derivatives of the entropy function with respect to the elements of the field, that is, $A = \partial_z s$. Consequently, matrix M can be written as

$$M(z) = \frac{\partial^2 s}{\partial z_i \partial z_j} < 0. \tag{6}$$

The boundary $\mathcal{B}$ is assumed to be partitioned into a number of sets of positive measure Γ_c, Γ, and Γ' such that $\mathcal{B} = \Gamma_c \cup \Gamma \cup \Gamma'$. Γ_c refers to that part of the boundary through which material flows with velocity $\boldsymbol{v}$ (convective flow). This boundary will be divided into two disjoint sets Γ_c^+ and Γ_c^- according to the following inequalities

$$\boldsymbol{v} \cdot n(\Gamma_c^-) \le 0 \tag{7}$$

$$\boldsymbol{v} \cdot n(\Gamma_c^+) \ge 0, \tag{8}$$

where $\boldsymbol{n}(\Gamma_i)$ is a unit vector associated to Γ_i and pointing outwards. Equations 7 and 8 define that part of the boundary where material flows enter and leave, respectively. Second-order boundary conditions of the form

$$\eta \frac{dA}{dn} = -h[A - \pi(t)] \tag{9}$$

are considered on Γ, with $\pi(t)$ being an arbitrary function of time that can be used for manipulation. Finally, Γ' denotes that region of the boundary through which no mass or energy fluxes cross, thus

$$\eta \frac{dA}{dn} = 0. \tag{10}$$

Since we want to control the class of systems just described with respect to a stationary reference field z^*, we rewrite Eq. 3 in deviation form with respect to the reference, so that

$$\bar{z}_t + \nabla(\boldsymbol{v}\bar{z} + \bar{J}) = \bar{\sigma}, \tag{11}$$

where the overbar notation stands for deviations from the stationary reference. Boundary conditions on $\mathcal{B}$ then become

$$\bar{z} = 0 \quad \text{on} \quad \Gamma_c^- \tag{12}$$

$$\eta \frac{d\bar{A}}{dn} = 0 \quad \text{on} \quad \Gamma' \tag{13}$$

and

$$\eta \frac{d\bar{A}}{dn} = -h(\bar{A} - \bar{\pi}) \tag{14}$$

on Γ. For stability purposes, we restrict the class of possible control actions $\bar{\pi}$ to those satisfying an inequality of the form

$$-k_1 \bar{A}^T \bar{A} \le \bar{A}^T \bar{\pi}(t) \le -k_2 \bar{A}^T \bar{A} \tag{15}$$

for some arbitrary constants k_1, $k_2 \ge 0$. Note that this condition reduces the type of input functions $\bar{\pi}$ to either $\bar{\pi} = 0$ or those inducing negative feedback in the system.

The following structures for the production σ are considered:

$$\sigma(z,u) = g(z) + u_1(p,t) \tag{16}$$

$$\sigma(z,u) = g(z)u_2(t), \tag{17}$$

where $g(z)$ is a nonlinear function of the field and $u_1(p, t)$, $u_2(t)$ are functions that can be manipulated externally. In Eq. 16, u_1 can depend on the spatial coordinates p and time, while u_2 in Eq. 17 is assumed to depend exclusively on time. The structures selected are representative of interesting distributed parameter systems we intend to control in a robust way. For instance, Eq. 16 can be used in tubular chemical reactors to describe reaction rate or heat of reaction. In the first case, u_1 would represent the addition of chemical components, while in the second case, it should be interpreted as the rate of heat exchanged (both quantities taken along the axial direction). It is noted that in order to be able to manipulate u_1 externally, a mixing condition in the radial direction must be satisfied. Equation 17 is employed to describe the rate of heat dissipated by electric or electromagnetic fields such as ohmic or microwave heating. In this case $u_2(t)$ must be interpreted as the electric/electromagnetic power supplied and $g(z)$ as a nonlinear function describing the local effect of that field on the system. Relevant examples of this class can be found in, among areas, food processing and materials technology (see, for instance, Fukushima et al., 1990; Ayappa et al., 1991; Beale and Li, 1997). The structure of the production terms—Eqs. 16 and 17—motivates the following classification.

Type 1 Systems. Given a reference u_1^* and $z^* \in Z^*$, the production (Eq. 16) around (u_1^*, z^*) takes the form

$$\bar{\sigma} = [g(z) - g(z^*)] + u_1 - u_1^*. \tag{18}$$

Type 2 Systems. Given a reference u_2^*, $z^* \in \mathcal{Z}^*$, the production (Eq. 17) around (u_2^*, z^*) takes the form

$$\bar{\sigma} = [g(z) - g(z^*)]u_2 + g(z^*)(u_2 - u_2^*). \tag{19}$$

Equations 18 and 19 can be considered as particular cases of the more general structure

$$\bar{\sigma} = \bar{g}\upsilon(t) + \gamma(p)\bar{u}, \tag{20}$$

where u stands for the control, and $\upsilon(t)$, $\gamma(p)$ are positive functions. For the cases under consideration, these are

$$\text{Type 1 } \upsilon(t) = 1 \qquad \gamma(p) = 1$$

$$\text{Type 2 } \upsilon(t) = u_2 \qquad \gamma(p) = g(z^*).$$

The following assumptions are imposed on the class of distributed process systems (in short DPS class) described by Eq. 11 with boundary (Eqs. 12–14) and production of the form in Eq. 20:

Assumption 1. There exists a finite domain, $\mathcal{V}_D$, which we will refer to as the domain of dissipation, where the following inequality holds

$$\eta < \bar{X}, \bar{X} >_{\mathcal{V}_D} + < \bar{A}, \bar{\sigma} >_{\mathcal{V}_D} \geq 0,$$

with $<.,.>_{\mathcal{V}}$ defined as in Eq. 2 and η being a positive constant. Physically, this condition presumes the existence of a universe divided into portions where fluxes compensate for the rate of production. Such a description is consistent with the thermodynamic universe in the sense that properties remain bounded. This is shown in the next section by using a convex extension b derived from thermodynamic arguments and summarized in the Appendix.

Assumption 2. Let (A^*, J^*) be an arbitrary reference, then every pair (X, J) satisfies

$$<(X - X^*),(J - J^*)>_{\mathcal{V}} \geq \eta <(X - X^*),(X - X^*)>_{\mathcal{V}}. \tag{21}$$

This condition formalizes a well-known phenomenon: an increase in the thermodynamic forces is compensated by an increase in the thermodynamic fluxes. The validity of this assumption is evident in the so-called linear branch (deGroot and Mazur, 1962) where L, although possibly a function of the position, is independent of the field. In this context, constant η in inequality 21 can be defined as

$$\eta \leq \min_z \inf_i \{\lambda_i(L)\},$$

where λ_i represents the i eigenvalue of matrix L.

Function $g(z)$ in Eq. 16 or Eq. 17 is assumed to be Lipschitz continuous, as it is standard in the theory of parabolic operators (Smoller, 1983). This condition is written as follows.

Assumption 3. Consider some reference z^* and a set $Z^* = \{z/|z - z^*| < c\}$, with c being constant. Then, there exist positive constants k_1 and k_2 such that for any $z \in Z^*$

$$-k_1(A - A^*)^T(A - A^*) \leq [g(z) - g(z^*)]^T(A - A^*) \leq -k_2(A - A^*)^T(A - A^*). \tag{22}$$

Note that this assumption (Eq. 22) can also be written as

$$<(A - A^*),(g - g^*)>_{\mathcal{V}} \geq -K <(A - A^*),(A - A^*)>_{\mathcal{V}}, \tag{23}$$

with constant K defined as $K = \sup_{\mathcal{V}} \sup_{Z^*}(k_1, k_2)$.

Passivity Conditions

This section contains the main result of the article, namely the conditions under which the class of distributed process systems described earlier become passive, that is, satisfy an inequality of the form of Eq. 1. In order to provide a rigorous justification, we present some previous results regarding bounding properties of the field and the role played by the spatial domain on the dynamics of the field.

Lemma 1. The field $\bar{z}$ is bounded in the dissipation domain $\mathcal{V}_D$, that is, $\bar{z} \in L^2(\mathcal{V}_D)$.

Proof. Using b as defined in the Appendix, we have that $b_t = -\bar{A}^T\bar{z}_t$, since z^* was chosen stationary. Substituting Eq. 11 in this expression, we get

$$b_t = \bar{A}^T\nabla(\mathcal{V}\,\bar{z}) + \bar{A}^T\nabla\bar{J} - \bar{A}^T\bar{\sigma}.$$

Integrating over $\mathcal{V}_D$ and using $B = \int_{\mathcal{V}_D} b(z, z^*)\, d\mathcal{V}$, we obtain

$$B_t = < \bar{A}, \nabla(\mathcal{V}\,\bar{z}) >_{\mathcal{V}_D} + < \bar{A}, \eta\frac{d\bar{A}}{dn} >_{\mathcal{B}_D} - < \bar{X}, \bar{J} >_{\mathcal{V}_D} - < \bar{A}, \bar{\sigma} >_{\mathcal{V}_D}$$

$$B_t \leq -\int_{\mathcal{B}_D} b\mathbf{v}n\, d\mathcal{B} - \eta < \bar{X}, \bar{X} >_{\mathcal{V}_D} - < \bar{A}, \bar{\sigma} >_{\mathcal{V}_D}, \tag{24}$$

where the following relations have been used

$$< \bar{A}, \nabla(\mathbf{v}\bar{z}) >_{\mathcal{V}_D} = - < \sum_{k=1}^{3}\left(\frac{\partial b}{\partial \bar{z}}\right)\frac{\partial(\mathbf{v}\bar{z})}{\partial x_k} >_{\mathcal{V}_D} = -\int_{\mathcal{B}_D} b\mathbf{v}n\, d\mathcal{B} \tag{25}$$

$$< \bar{A}, \nabla\bar{J} >_{\mathcal{V}_D} = < \bar{A}, \eta\frac{d\bar{A}}{dn} >_{\mathcal{B}_D} - < \bar{X}, \bar{J} >_{\mathcal{V}_D} \leq -\eta < \bar{X}, \bar{X} >_{\mathcal{V}_D}. \tag{26}$$

Note that in deriving Eq. 26 use has been made of boundary conditions (Eq. 14) and inequality 15. From Eqs. 7 and 8, and the fact that $z = z^*$ on Γ_c^-, $b = 0$ at the input convective boundary. Consequently, the righthand side term in Eq. 25 is negative. From Assumption 1 we then conclude that $B_t \leq 0$ and $B(t) \leq B(t = 0)$. This, together with the properties for b and the compactness of the domain $\mathcal{V}_D$ implies $\bar{z} \in L^2(\mathcal{V}_D)$ (that is, the field is bounded).

Lemma 2. For $\mathcal{V} \subset \mathcal{V}_D$, the DPS class accepts an evolution inequality of the form

$$B_t \leq -\mu_1\eta < \bar{A}, \bar{A} >_{\mathcal{V}} - < \bar{A}, \bar{\sigma} >_{\mathcal{V}}, \tag{27}$$

where μ_1 is a positive parameter that depends on the size of the domain $|\mathcal{V}|$. In fact, μ_1 increases as the size of the domain decreases.

Proof. From inequality 24 and the negative condition in Eq. 25, we have that

$$B_t \leq \eta < \bar{A}, \frac{d\bar{A}}{dn} >_{\mathcal{B}} - \eta < \bar{X}, \bar{X} >_{\mathcal{V}} - < \bar{A}, \bar{\sigma} >_{\mathcal{V}}. \quad (28)$$

Since $\mathcal{V} \subset \mathcal{V}_D$, $\bar{z} \in L^2(\mathcal{V}_D)$ (Lemma 1) and therefore $\bar{A}$ accepts an orthogonal series expansion of the form

$$\bar{A} = \sum_{j=1}^{\infty} c_j \varphi j, \quad (29)$$

where each φ_j satisfies the following equations

$$-\Delta \varphi_j = \mu_j \varphi_j \quad (30)$$

$$< \varphi_i, \varphi_j >_{\mathcal{V}} = \delta_{ij}, \quad (31)$$

with δ_{ij} the Kronecker delta. Parameters μ_j are called the eigenvalues associated to eigenfunctions φ_j and are ordered as $\mu_i < \mu_j$ for any $i < j$. Using Eq. 29 and Green's formula, we get

$$\eta < \bar{X}, \bar{X} >_{\mathcal{V}} = -\eta < \bar{A}, \Delta \bar{A} >_{\mathcal{V}} + \eta < \bar{A}, \frac{d\bar{A}}{dn} >_{\mathcal{B}} \quad (32)$$

$$< \bar{A}, \Delta \bar{A} >_{\mathcal{V}} \leq -\mu_1 < \bar{A}, \bar{A} >_{\mathcal{V}}. \quad (33)$$

These relations are also known as Poincaré inequalities (Smoller, 1983). Substituting Eq. 33 in Eq. 32, we obtain

$$\eta < \bar{A}, L\frac{d\bar{A}}{dn} >_{\mathcal{B}} - \eta < \bar{X}, \bar{X} >_{\mathcal{V}} \leq -\mu_1 \eta < \bar{A}, \bar{A} >_{\mathcal{V}},$$

which combined with Eq. 28 leads to the desired inequality

$$B_t \leq -\mu_1 \eta < \bar{A}, \bar{A} >_{\mathcal{V}} - < \bar{A}, \bar{\sigma} >_{\mathcal{V}}.$$

For boundary conditions of the form

$$\eta \frac{d\bar{A}}{dn} = -\alpha \bar{A},$$

where α is a positive constant, the parameter μ_1 is known to depend continuously on the size of the domain $|\mathcal{V}|$. In fact, μ_1 increases as the size of the domain decreases (Courant and Hilbert, 1937). It turns out that this is also the case for boundary conditions of the form in Eq. 14 provided that inequality 15 holds. This we show by comparing μ_1 with the principal eigenvalue of the auxiliary system

$$-\Delta \varphi_1' = \mu_1' \varphi_j' \quad (34)$$

$$\eta \frac{d\varphi_1'}{dn} = -\alpha \varphi_1', \quad (35)$$

where both Eq. 34 and Eq. 35 are defined on the domain $(\mathcal{V}, \mathcal{B})$. First note that μ_1 in Eq. 30 is the extrema (maximum) of the functional

$$Q(\varphi) = \frac{< \nabla\varphi, \nabla\varphi >_{\mathcal{V}} - < \varphi, \eta\,(d\varphi/dn) >_{\mathcal{B}}}{< \varphi, \varphi >_{\mathcal{V}}} \quad (36)$$

subject to the conditions in Eq. 31 (Courant and Hilbert, 1937). Using Eqs. 14 and 15, we have that

$$< \bar{A}, \eta \frac{d\bar{A}}{dn} >_{\mathcal{B}} \leq -h(1 + k_2) < \bar{A}, \bar{A} > \mathcal{B}.$$

Since $\bar{A}$ can be expanded in the form of Eq. 29, we also have that

$$< \varphi, \eta \frac{d\varphi}{dn} >_{\mathcal{B}} \leq -h(1 + k_2) < \varphi, \varphi >_{\mathcal{B}}.$$

Substituting this inequality in Eq. 36, we obtain

$$Q(\varphi) \geq \frac{< \nabla\varphi, \nabla\varphi >_{\mathcal{V}} + h(1 + k_2) < \varphi, \varphi >_{\mathcal{B}}}{< \varphi, \varphi >_{\mathcal{V}}}.$$

The righthand term in the previous inequality achieves a maximum for φ' satisfying Eqs. 34 and 35, with $\alpha = h(1 + k_1)$. Thus $\mu_1 \geq \mu_1'$ and the result follows since μ_1' in Eqs. 34 and 35 increases as the size decreases.

Inequality 27 in Lemma 2 has an immediate physical interpretation: the evolution of the field around the reference will depend on the net effect of two opposite contributions we refer to as dissipative and nondissipative effects. The first term on the righthand side collects all sources of dissipation due to fluxes in the system and promotes, with its negative sign, the decrease of the instant value of B. The second term includes production and contributes, with its positive sign, to the increase in the instant value of B. To clarify this point, let us consider an open-loop experiment with constant inputs. Under such a condition, $\bar{u} = 0$ and Eq. 20 reduces to $\bar{\sigma} = \bar{g}\upsilon(t)$. By Assumption 3, inequality 22, we have that

$$< \bar{A}, \bar{\sigma} >_{\mathcal{V}} = < \bar{A}, \bar{g} >_{\mathcal{V}} \upsilon(t)$$

$$- < \bar{A}, \bar{\sigma} >_{\mathcal{V}} \geq \alpha < \bar{A}, \bar{A} >_{\mathcal{V}} \geq 0,$$

with α chosen as the minimum over all possible values of k_2 in the domain. Since B defines a distance for the states from their reference (see the Appendix), when dissipation predominates over nondissipative effects, the righthand side of the inequality becomes negative and the states evolve closer to the reference, thus precluding stability. On the other hand, instability phenomena can emerge when the sign condition is not met. In this context, controllers may be envisioned as devices that cooperate with fluxes to enhance dissipation and thus promote convergence of the states. As we will see, passivity makes such cooperation possible.

Proposition 1. For any bounded input, there exists a size $|\mathcal{V}| \leq |\mathcal{V}_D|$ which makes the DPS class passive. Such a domain is referred to as the passive domain.

Proof. Using Lemma 2, inequality 27 and Eq. 20, we have

$$B_t \leq -\mu_1 \eta < \bar{A}, \bar{A} >_{\mathcal{V}} - < \bar{A}, \bar{\sigma} >_{\mathcal{V}} \quad (37)$$

$$< \bar{A}, \bar{\sigma} >_{\mathcal{V}} = \upsilon(t) < \bar{A}, \bar{g} >_{\mathcal{V}} + < \bar{A}, \gamma(p)\bar{u} >_{\mathcal{V}}. \quad (38)$$

From Assumption 3,

$$< \bar{A}, \bar{g} >_{\mathcal{V}} \geq -K < \bar{A}, \bar{A} >_{\mathcal{V}}. \quad (39)$$

Combining Eqs. 37 to 39, and defining $\bar{\phi} = -\gamma(p)\bar{u}$ as the control input, we obtain

$$- < \bar{A}, \bar{\sigma} >_{\mathcal{V}} \leq K\upsilon(t) < \bar{A}, \bar{A} >_{\mathcal{V}} + < \bar{A}, \bar{\phi} >_{\mathcal{V}}$$

$$B_t \leq -(\mu_1 \eta - K\upsilon) < \bar{A}, \bar{A} >_{\mathcal{V}} + < \bar{A}, \bar{\phi} >_{\mathcal{V}}, \quad (40)$$

where μ_1 is a continuous function of the size of the spatial domain that increases as the size of the domain decreases (Lemma 2). Consequently, for υ bounded, we can always find a domain with size $|\mathcal{V}|$ such that $\mu_1(|\mathcal{V}|)\eta - K\upsilon > 0$. This is obviously satisfied for Type 1 systems. Type 2 systems also satisfy this condition, provided that the input u_2 is bounded. For this domain, we integrate Eq. 40 in time, so that

$$B(t+\tau) - B(t) \leq \int_t^{t+\tau} < \bar{A}, \bar{\phi} >_{\mathcal{V}} ds. \quad (41)$$

Since B is bounded from below (see the Appendix), it is an appropriate storage, and the result follows by comparing Eq. 41 and Eq. 1 with $y = \bar{A}$ and $w = \bar{\phi}$.

Passive Control Design

Two main consequences for control design emerge from Proposition 1: by properly choosing inputs and outputs in Eq. 41, the states can be made to converge exponentially fast toward their reference. Passivity also allows the design of observers capable of reconstructing the infinite dimensional state at arbitrary precision from a limited number of measurements. These two properties will lead to efficient control schemes that, with very limited information about the system, will preserve stability for a wide range of conditions. A formal discussion of these consequences is given next.

Exponential stabilization of distributed process systems

Consequence 1. The passive DPS-class is exponentially stable for inputs such that

$$< \bar{A}, \bar{\phi} >_{\mathcal{V}} \leq 0. \quad (42)$$

Proof. From the differential inequality 40 and Eq. 42, we have that

$$B_t \leq -\theta < \bar{A}, \bar{A} \geq_{\mathcal{V}},$$

with $\theta = \mu_1 \eta - K\upsilon > 0$ (Proposition 1). From result 3 in Theorem A1 (Appendix),

$$< \bar{A}, \bar{A} >_{\mathcal{V}} \geq \beta B$$

$$B_t \leq -\theta\beta B.$$

From Gronwall-Bellman Lemma (Slotine and Li, 1991), $B(t) \leq B(0)\exp(-\theta\beta t)$, and the result follows, that is, $|z| \to 0$ at exponential rate.

Parameter θ contains extremely useful information to be considered in guiding process design decisions. Parameters η and K are the system's parameters that will tell us how fast heat or chemical species diffuse through the domain, and the rate of property production (heat produced or chemicals formation). These properties can also be affected by the field—equivalently, by its intensive counterparts (temperature, concentrations, etc.)—thus determining the range of operation. On the other hand, parameters μ_1 and υ depend on design considerations such as the volume of the process (or, in a more general context, the size of the domain) or the maximum allowed input (in Type 2 systems $\upsilon = u_2$). In order to construct a stable, easily controllable plant, design decisions should be taken such that θ remains positive. Of course, this condition must also be confronted with economic objectives as well.

Since $\bar{\phi} = 0$ satisfies inequality 42, the passive DPS-class is open-loop stable. Note also that for systems of Type 2 with $u_2 = \bar{u}_2 = 0$, the preceding result holds trivially for any domain $\mathcal{V}$. In other words, any domain is a domain of dissipation. This result coincides with what Ydstie and Alonso (1997) define as purely dissipative systems. Inequality 42 may also include as inputs linear and/or nonlinear maps $\bar{\phi}(\bar{A})$ or $\bar{\phi}(\bar{y})$, where $\bar{y} = \int_{\mathcal{V}} \bar{A}\, d\mathcal{V}$ is directly related to the mass and energy inventories in the domain. Their convergence properties are stated in the following two results:

Consequence 2. If maps of the form $\mathcal{K}: \bar{A} \to \bar{\phi}$ exist such that

$$< \bar{A}, \bar{\phi} >_{\mathcal{V}} \leq -\alpha < \bar{A}, \bar{A} >_{\mathcal{V}} + < \bar{A}, \bar{\phi}' >_{\mathcal{V}} \quad (43)$$

for some $\alpha > 0$, then passivity is preserved for any spatial domain.

Proof. Using Proposition 1 and substituting Eq. 43 on the righthand side of the passivity inequality 41, we obtain

$$B(t+\tau) - B(t) \leq$$
$$-\int_t^{t+\tau} \left[(\mu_1 \eta - K\upsilon + \alpha) < \bar{A}, \bar{A} >_{\mathcal{V}} - < \bar{A}, \bar{\phi}' >_{\mathcal{V}} \right] ds.$$

Choosing $\alpha > K\upsilon - \mu_1 \eta$, the inequality becomes

$$B(t+\tau) - B(t) \leq \int_t^{t+\tau} < \bar{A}, \bar{\phi}' >_{\mathcal{V}} ds$$

and the result follows.

This result can be linked to the notion of stabilization by high gain control. $\mathcal{K}$ maps can be easily constructed for well-mixed processes where $\overline{A}$ and $\overline{\phi}$ take uniform values (values independent of the spatial position). In this case, the input can be constructed as

$$\overline{\phi} = -\alpha\overline{A} + \overline{\phi}'.$$

Note that, according to Consequence 1, the existence of these maps ensures exponential stability under inputs satisfying Eq. 42. In distributed systems these maps usually involve combinations of different types of inputs. For instance, $\mathcal{K}$ maps can be constructed as the combination of different sources of heating/cooling devices. When complete mixing does not exist, it is still possible to stabilize the process for spatial domains smaller than the domain of dissipation. Performance can then be improved through inventory control as shown next.

Consequence 3. Let $\overline{\phi}$ in Eq. 41—equivalently, $\overline{\phi}'$ in Eq. 43—be uniform inputs (that is, independent of position), and define a map (control law) $\mathcal{I}$: $\bar{y} \rightarrow \overline{\phi}$ such that

$$\bar{y} = \int_{\mathcal{V}} \overline{A}\, d\mathcal{V}$$

$$< \overline{A}, \overline{\phi} >_{\mathcal{V}} \leq -\alpha' \bar{y}^T \bar{y},$$

then $|\bar{y}| \rightarrow 0$ and $|\bar{z}| \rightarrow 0$ at an exponential rate.

Proof. From Proposition 1—inequality 40, or Consequence 2—inequality 43, we have, after applying map $\mathcal{I}$, that

$$B_t \leq -\theta < \overline{A}, \overline{A} >_{\mathcal{V}} - \alpha' \bar{y}^T \bar{y},$$

with $\theta = \mu_1 \eta - K\upsilon + \alpha > 0$ and $\alpha \geq 0$. Since the second term on the righthand side is negative,

$$B_t \leq -\theta < \overline{A}, \overline{A} >_{\mathcal{V}},$$

and the result follows as in Consequence 1.

Note that for inputs of the form $\overline{\phi} = -\gamma(p)\bar{u}(t)$, Consequence 3 applies by redefining y as $y = \int_{\mathcal{V}} \gamma(p)\overline{A}\, d\mathcal{V}$. This result demonstrates feedback stabilization of the field by finite dimensional controllers requiring inventories related to mass and energy as measurements. However, a question of obvious practical interest remains: How to reconstruct the inventory $\bar{y}$ from the measurements available (usually taken at a finite number of positions on the boundary)? An answer to this question is proposed next for linear parabolic operators, based on their time-scale separation property (Chen and Chang, 1992).

State reconstruction through passive observers

Let us assume that fluxes J are linear and rewrite Eq. 11 as

$$\bar{z}_t = -\mathfrak{L}(\bar{z}) + \bar{\sigma}(z, u; z^*, u^*). \tag{44}$$

In addition let us consider a general *dummy* linear system of the form:

$$\hat{\bar{z}}_t = -\mathfrak{L}(\hat{\bar{z}}) + K\upsilon\hat{\bar{z}} + \gamma(p)\bar{u}, \tag{45}$$

where υ, $\gamma(p)$ and K are defined as in Eqs. 20 and 23, respectively.

Here $\mathcal{V} \in \mathcal{V}_D$ so that Lemma 1 applies and $\bar{z}$ and $\hat{\bar{z}}$ are bounded functions accepting an orthogonal series expansion of the form given in the proof of Lemma 2 for its dual $\overline{A}$—Eq. 29. In the light of this argument, we multiply both sides of Eqs. 44 and 45 by eigenfunctions φ_j, for $j = 1, \ldots, n$, and integrate on the domain $\mathcal{V}$ to obtain a set of ordinary differential equations

$$\dot{c} = -\Lambda c + \upsilon F + \bar{u}G \tag{46}$$

$$\dot{\hat{c}} = -\Lambda\hat{c} + K\upsilon I\hat{c} + \bar{u}G, \tag{47}$$

which capture the slowest n-dimensional dynamics of the original PDE systems (Eqs. 44 and 45, respectively) (Chen and Chang, 1992). For diffusive processes $\dot{\Lambda} = \mathrm{diag}(\mu_j)$. The terms F, $G \in R^n$ are of the form

$$F = \left[\ldots, <\varphi_j, \bar{g}>_{\mathcal{V}}, \ldots\right]^T$$

$$G = \left[\ldots, <\varphi_j, \gamma(p)>_{\mathcal{V}}, \ldots\right]^T.$$

In addition, let us consider an m-dimensional vector ξ of measurements taken at particular locations p_i ($i = 1, \ldots, m$) and its corresponding estimation vector $\hat{\xi}$. Both measurements and states are related by Eq. 29, which in its truncated form leads to

$$\hat{\xi}_1 = \sum_{j=1}^{n} \hat{c}_j \varphi_j(p_1)$$

$$\hat{\xi}_m = \sum_{j=1}^{n} \hat{c}_j \varphi_j(p_m).$$

Systems of the form of Eqs. 46 and 47 are observable with respect to outputs ξ, at least when measurements are taken at the boundary (Waldorff et al., 1998). Thus, if the true plant were linear, a Luemberger observer could be used to reconstruct at exponential rate the state of the system (Kailath, 1980). In our case, the observer would be of the form

$$\dot{\hat{c}} = -\Lambda\hat{c} + K\upsilon I\hat{c} + \bar{u}G + \omega\left(\xi - \hat{\xi}\right), \tag{48}$$

with $\omega \in R^{n \times m}$ being a gain matrix to be chosen later on. The same could be said for a nonlinear system of the form of Eq. 46, were the nonlinear terms precisely known. When the plant is neither linear nor completely known, Eq. 48 can still provide estimates that will make $\hat{\bar{z}}$ converge asymptotically to the true state $\bar{z}$, at least in the time scale of the slow dynamics. As we show next, such a remarkable property is a consequence of the system being passive.

Proposition 2. Let Proposition 1 holds and u be linear piecewise constant on each interval $[t_i, t_i + T]$ for $t_i, T > 0$, then the error $e = c - \hat{c}$ satisfies

$$|e| \le |e(0)| \exp\left(-\frac{\lambda t}{2}\right) + M(t;\tau,\lambda),$$

where $\tau = (\mu_1 - K\upsilon)^{-1}$, λ is an arbitrarily selected constant and $\lim_{t\to\infty}(M(t;\tau,\lambda) = 0$.

Consequently, $\hat{c} \to c$ asymptotically.

Proof. Let us first rewrite Eq. 48 as

$$\dot{\hat{c}} = \Lambda\hat{c} + K\upsilon I\hat{c} + \bar{u}G + \omega\Pi(c - \hat{c}), \tag{49}$$

with Π being

$$\Pi = [\pi_1 \vdots \pi_i \vdots \pi_m]^T$$

$$\pi_i = [\varphi_1(p_i), \ldots, \varphi_j(p_i), \ldots, \varphi_n(p_i)].$$

Combining Eqs. 46 and 49, we get

$$\dot{e} = -(\Lambda - K\upsilon I)e + \upsilon F - K\upsilon Ic - \omega\Pi e.$$

Note that since the system is observable, ω can be selected so that $\Lambda - K\upsilon I + \omega\Pi = A$, for A being an arbitrary matrix. In particular, we can choose A to be a positive-definite matrix with λ being its smallest eigenvalue. Let us now define a function $W = 1/2 e^T e$. Differentiating W with respect to time and combining it with the previous equation, we obtain

$$\frac{dW}{dt} = -e^T Ae + e^T(\upsilon F - K\upsilon Ic). \tag{50}$$

The second term on the righthand side can be bounded as

$$e^T(\upsilon F - aIc) \le m|e| \int_{\mathcal{V}} |\bar{g} - a\bar{z}|\, d\mathcal{V}, \tag{51}$$

where m is a positive constant. Since the system is passive and u is piecewise constant on intervals $[t_i, t_i + T]$, $\bar{u} = 0$ on each interval and the field evolves so that $|\bar{z}| \le |\bar{z}(0)| \exp(-t/\tau)$ for some positive constant $\tau^{-1} > 0$ (Consequence 1). Combining this bound with inequality 51 leads to

$$e^T(\upsilon F - aIc) \le h\sqrt{W}\exp(-t/\tau)$$

for some positive constant h. Introducing this bound in inequality 50, we get

$$\frac{dW}{dt} \le -\lambda W + h\sqrt{W}\exp(-t/\tau).$$

Using transformations $w = \sqrt{W}$ and $r = w \exp(\lambda t/2)$, the explicit solution for the previous inequality is derived as follows

1. Differentiate w to obtain

$$\frac{dw}{dt} + \frac{\lambda}{2}w \le \frac{h}{2}\exp(-t/\tau).$$

2. Premultiply both sides of the inequality by $\exp(\lambda t/2)$ and use transformation $r = w\exp(\lambda t/2)$ so that

$$\frac{dr}{dt} \le \frac{h}{2}\exp\left[\left(\frac{\lambda}{2} - \frac{1}{\tau}\right)t\right].$$

3. Integrate in time both sides of the inequality, to obtain

$$w(t)\exp(\lambda t/2) - w(0) \le \frac{h}{2}\int_0^t \exp\left[\left(\frac{\lambda}{2} - \frac{1}{\tau}\right)s\right]ds$$

$$w(t) \le w(0)\exp(-\lambda t/2) + \frac{h}{2}\int_0^t \exp\left[-\frac{\lambda t}{2} + \left(\frac{\lambda}{2} - \frac{1}{\tau}\right)s\right]ds.$$

Since $w = |e|$, we then have

$$|e| \le |e(0)|\exp\left(-\frac{\lambda t}{2}\right) + M(t;\tau,\lambda)$$

$$M(t;\tau,\lambda) = \frac{h}{\lambda - 2\tau^{-1}}\left[\exp(-t/\tau) - \exp\left(-\frac{\lambda t}{2}\right)\right].$$

Consequently, $\hat{c} \to c$ asymptotically.

According to this result, the field $\bar{z}$ can be reconstructed at arbitrary precision from measurements at the boundary. Precision is determined by the dimension of the observer (Eq. 49), which captures the low-dimensional dynamics. The reconstruction then proceeds by combining the estimates $\hat{c}$ with the corresponding eigenfunctions, so that

$$\bar{z}(p,t) \simeq \sum_{j=1}^{n} \varphi_j(p)\hat{c}_j. \tag{52}$$

Some Illustrative Examples

The results and discussion presented in the last section suggest a general control structure for distributed process systems, as depicted in Figure 1. Controllers $\bar{\pi}(\bar{A})$, satisfying

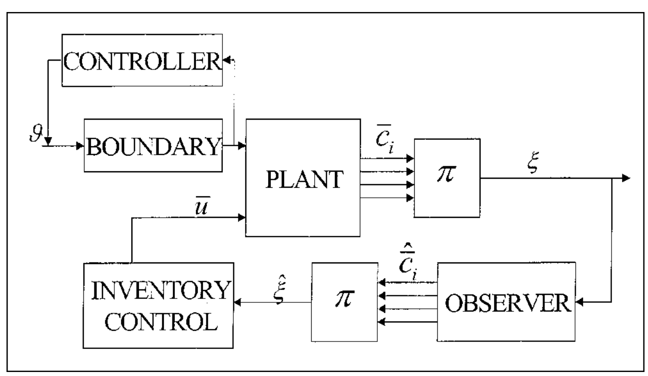


Figure 1. Proposed control structure.

For illustration purposes, the system has been split into boundary and plant.

inequalities 15, would operate on the boundary in response to changes of the state around some preselected setpoint. Alternatively, π could be kept at its reference π^* by cascading a controller on the external manipulated variable ϑ. Such alternative, represented in Figure 1 will be discussed in more detail in the microwave thermal processing example below. In order to implement the passive control designs discussed in the previous section, conditions must be found so that Proposition 1 holds. For systems of Type 1, such as tubular chemical reactors, passivity constrains the domain of reaction and operating conditions to those that make the term $\mu_1(|\mathcal{V}|)\eta - K$ positive. Since from Lemma 2 the principal eigenvalue μ_1 is a decreasing function of the volume of the domain, a critical size can always be found for a given diffusivity and kinetics. For Type 2 systems, an upper bound on u_2 can also be employed as an additional parameter to preserve passivity.

The appropriate stabilizing control law will be selected in accordance with Consequence 1 among those satisfying inequality 42. In the event of perfect mixing in some direction (such as Type 1 systems), maps of the form given in Consequence 2 can be constructed to ensure passivity and exponential stabilization for any spatial domain. Alternatively, inventory control could be employed on the passive domain, as proposed in Consequence 3. The field $\bar{z}$, or its dual $\overline{A}$, required by these control maps is reconstructed from passive observers of the form of Eq. 48. Next, these ideas are illustrated on the basis of two examples representative of the class of systems considered.

Passivity and exponential stabilization in tubular reactors

The control problem reduces to finding appropriate (passive) maps, as shown in Consequence 1, between the manipulated variable $\bar{u}$ and the field $\bar{z}$ or its dual $\overline{A}$. Two ways of constructing such functions are possible: the *hardware* and the *software* approach. In the first case, feedback control results from process systems interconnections; in the second case, a computing device is employed to build up the required feedback. A crucial aspect to be considered in the software approach is to decide where to measure and where to act. In some case, the whole infinite dimensional state should be available that implies the search for method of state reconstruction on the basis of a finite number of measurements, such as those proposed in the previous section. Most practical control designs for process systems share both hardware and software approaches.

A common example of the hardware approach can be found in tubular reactors. In this case, the feedback function is constructed by adding a heating/cooling device (heat exchanger) with enough heat transfer area (see Figure 2a). This interconnection provides a control of the form $\bar{u}_1 = -Ua_V\bar{z}$, where U is the heat-transfer coefficient and a_V the heat-transfer area-volume ratio. The duality between $\overline{A}$ and $\bar{z}$ allows us to write an equivalent feedback function $\bar{u}_1 = U'a_V\overline{A}$, where U' in this representation is still positive. Since for Type 1 systems $\overline{\phi} = -\bar{u}_1$, the resulting map satisfies inequality 43 with $\alpha = U'a_V$ and $\overline{\phi}' = 0$ (Consequence 2). Substitution of this expression in Eq. 40 results in an inequality of the form

$$B_t \leq -(\mu_1\eta + U'a_V - K)\beta B,$$

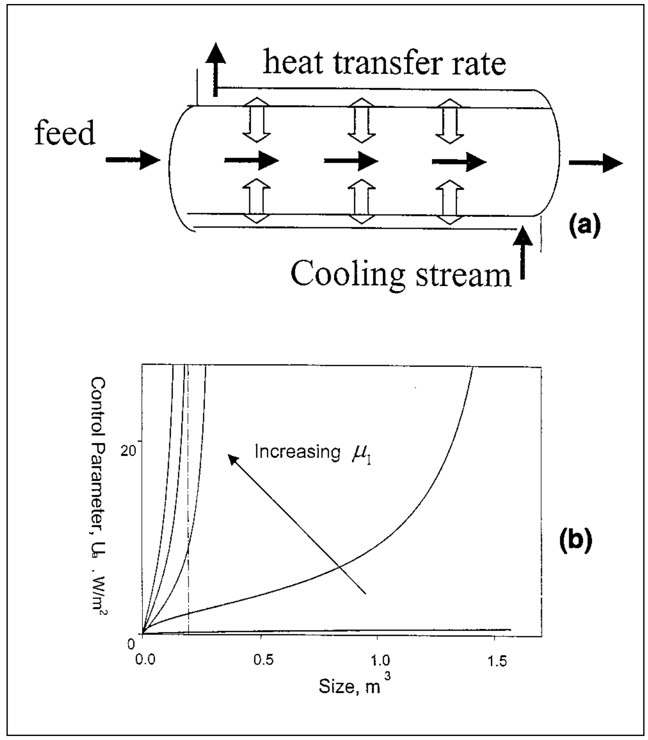


Figure 2. Passivity in tubular reactors.

(a) Tubular reactor with a heat exchanger; (b) principal eigenvalue for different heat-transfer parameters and domains. Continuous lines represent places where the principal eigenvalue is constant. Dotted straight line illustrates the asymptotic behavior of the principal eigenvalue, for a constant size, as the heat transfer coefficient increases.

where β is defined as in the Appendix (Theorem A1). Note that, at least at the design level, it is always possible to choose $U'a_V$ large enough so that $\mu_1\eta + U'a_V - K > 0$ for any given size and rate of heat produced (bounded by constant K). In this way, mixing in the radial direction allows the arbitrary selection of the rate of convergence by high gain control (heat-transfer area and/or heat transfer coefficient) as shown in Consequences 1 and 2. The field will converge in the norm defined by B as

$$B(t) \leq B(0)\exp\left(-\frac{t}{\tau}\right),$$

with $\tau^{-1} = (\mu_1\eta + U'a - K)\beta$. Such a condition is fundamental for designing stabilizing controllers in distributed systems. Note that in the case discussed, the setpoint would correspond to the temperature of the cooling stream, which could be selected by cascading an external controller acting on input flow or input temperature, for instance.

In practice, some energy/mol number distribution on the radial direction always exists. This weakens the mixing condition and prevents the use of arbitrarily high gains. To illustrate this point, let us consider a flow system with axial and radial energy dispersion, such as a chemical reactor where an exothermic reaction is taking place. Using the standard simplifications, temperature distribution inside the reactor follows an equation of the form

$$\frac{\partial \bar{T}}{\partial t} = -\upsilon \frac{\partial \bar{T}}{\partial z} + \alpha \left[\frac{\partial^2 \bar{T}}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \bar{T}}{\partial r} \right) \right] + \bar{g}(T, T^*),$$

where υ and α are appropriate dimensionless parameters and $\bar{g}$, the generation term in deviation form with respect to a stationary reference. As shown in Figure 2a, the temperature of the system is controlled with an external cooling jacket. This heat exchanger behaves as a proportional controller acting on the energy flow (or equivalently, temperature) through the wall, in response to changes in the temperature difference between the outer and inner medium. Mathematically, this can be represented as

$$\left[\frac{d\bar{T}}{dn} \right]_{r=R} = -Ua\bar{T}.$$

Boundary conditions at the input and output are assumed to be of the form

$$\bar{T}_{z=0} = 0 \tag{54}$$

$$\left(\frac{d\bar{T}}{dn} \right)_{z=L} = 0. \tag{55}$$

The principal eigenvalue (μ_1) of the Laplacian operator for this case can be computed from Eqs. 34 and 35 with the corresponding boundary conditions (Eqs. 53–55). The values of μ_1 for different heat-transfer coefficients Ua_V and reactor sizes are plotted in Figure 2b. As can be seen from the figure, for any given reactor size there exists a finite limiting value $\mu^* = \lim_{Ua_V \to \infty} \mu_1$. Thus, passivity is restricted (see Proposition 1) to reactors with a size satisfying $\mu^* > K/\eta$ being this limit independent of the gain Ua_V.

Microwave thermal processing

As mentioned previously, microwave thermal processing is an interesting representative of Type 2 systems. In this way, it will be used to illustrate the control design methodology and performance. This process is presented in Figure 3a. Model and parameters have been taken from Ayappa et al. (1991). The material to be processed is modeled as a solid slab of 1 cm thickness with constant thermal diffusivity (the parameters used in the model are summarized in Table 1). Temperature distribution in the slab follows a Fourier-type equation with a generation term of the form

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} + g(T, x) P_w, \tag{56}$$

where P_w represents the incident microwave power supplied by the magnetron ($0 < P_w < 25$ W·cm^{-2}) and $g(T, x)$ is a nonlinear function describing that part of the power absorbed by the material. This term has been modeled by assuming temperature-dependent dielectric properties. Its explicit form is (Ayyapa et al., 1991)

$$g(T, x) = \frac{\pi f}{\rho C_p} \epsilon_0 k''(T) \cdot \exp[-2\beta(T) \cdot x]$$

$$\beta(T) = \frac{2\pi f}{c} \sqrt{\frac{k' \left(\sqrt{1 + \left(\frac{k''}{k'} \right)^2} - 1 \right)}{2}}.$$

Note that, according to these relations, the power absorbed by the product increases with temperature. Such a phenomenon is, in some instances, responsible of temperature

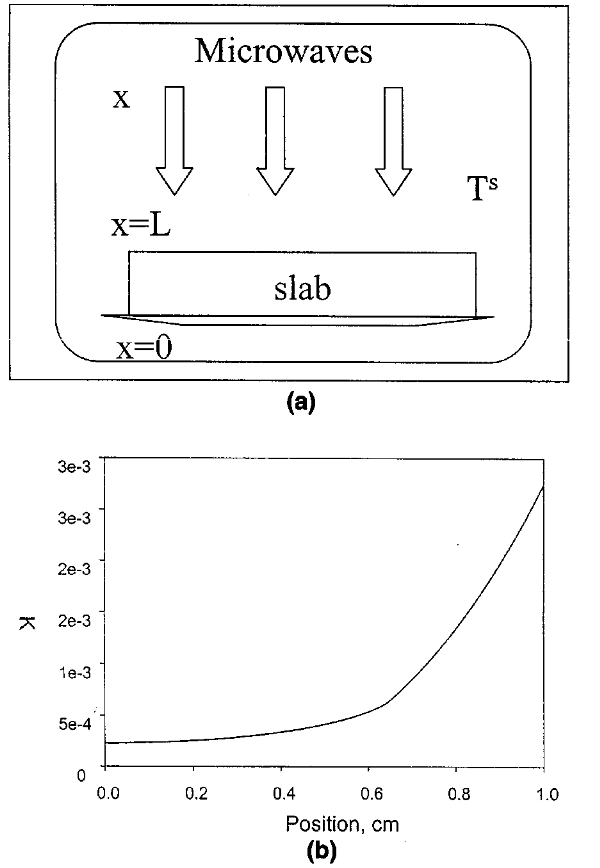


Figure 3. Control design in microwave processing.

(a) Microwave processing unit; (b) Values of $\sup_{Z^*}(k_1, k_2)$. (See inequalities 22 and 23 for different positions in the slab.

Table 1. Thermoelectric Properties for the Microwave Processing Model

f (frequency of incident radiation)	900 MHz
c (speed of light)	3×10^9 m·s^{-1}
ϵ_0 (free-space permittivity)	8.85×10^{-12} farad·m^{-1}
κ' (relative dielectric constant)	$82.23 - 0.1059\,T$
κ'' (relative dielectric loss)	$236.85 - 1.527T + 2.7277 \times 10^{-3} T^2$
η (thermal conductivity)	0.45 W·m^{-1}K^{-1}
h (heat-transfer coefficient)	2 W·m^{-2}·K^{-1}
ρ (density)	800 kg·m^{-3}
C_p (specific heat capacity)	2,850 J·kg^{-1}·K^{-1}

runaway [see, for instance, Fukushima et al. (1990)]. Other effects associated with the interaction of the electromagnetic field with matter include edge overheating or uneven heating (cold spots), thus limiting the range of application of this technology.

In addition, the following mixed-order boundary conditions (in agreement with physical considerations) are imposed on our system

$$T(t,0) = T^s \tag{57}$$

$$\left(\frac{dT}{dn}\right)_{x=L} = -\frac{h}{\eta}\left[T(t,L) - T^s\right], \tag{58}$$

with T^s being the oven temperature in the range 298 K $<$ $T^s <$ 373 K. The heat-transfer coefficient h and diffusivity η are summarized in Table 1. From the arguments in Proposition 1 it follows that this system is passive with respect to a temperature reference $T^*(x)$, satisfying

$$\alpha\frac{\partial^2 T^*}{\partial x^2} + g(T^*, x)P_w^* = 0,$$

provided that

$$P_w < \frac{\mu_1(L)\alpha}{K}, \tag{59}$$

where $\mu_1(L)$ refers to the principal eigenvalue of the Laplacian and K is chosen so that for $T \in Z^*$, with

$$Z^* = \{T / |T - T^*| < 80°C\}.$$

$K = \sup_{[0,L]} \sup_{Z^*}(k_1, k_2)$ (see inequalities 22 and 23). A plot of $k_{\max} = \sup_{Z^*}(k_1, k_2)$ for different positions in the slab is depicted in Figure 3b.

The control objective we choose consists of heating the material as uniformly as possible in spite of uncertainty in the material properties or initial temperature distribution, by acting on two possible manipulated variables: oven temperature T^s and power supply P_w. Other quality-type objectives could also be considered, such as minimizing vitamin degradation while achieving an acceptable C-value in the case of foods (Ohlsson, 1988), or the like in materials processing.

The control structure corresponds to that presented in Figure 1. Measurements are taken at the surface of the material (a low-cost infrared sensor can be used for that purpose) where second-order boundary conditions, Eq. 58, hold. State reconstruction is accomplished by a reduced-order observer of the form of Eq. 48. Note that we only allow a limited knowledge of Eq. 17. This consideration makes the control problem realistic, since from a practical point of view the nonlinear dependence of g with temperature is highly dependent on the material to be processed. On the other hand, its experimental determination is often a very time-consuming activity.

For this case, proportional and integral action were chosen for the control maps $\overline{\vartheta} = \varsigma(\overline{\pi})$ and $\overline{P}w = \phi(y)$. The first controller operates on the surrounding temperature, while the second one keeps the energy inventory under control. In agreement with Consequence 3, such a controller will ensure exponential stabilization of the internal temperature distribution. The tuning of the different controllers is carried out on the internal model control framework (Morari and Zafiriou, 1989) under the assumption of first-order input−output relationships. Control design is completed with low-pass filters of the form $f(s) = 1/(\lambda s + 1)$. The resulting controllers in velocity form become

Table 2. Different Control Parameters Employed in Experiments Depicted in Figures 4 and Figures 5.

	τ_p	k_p	λ_p	k_T	τ_T	λ_T
Figure 4	110	14.6	20	1	90	400
Figure 5	110	14.6	80	1	90	450

Subscripts p and T refer to power and surrounding temperature control, respectively.

$$P_w^{k-1} = P_w^{k-1} + \frac{\tau_p}{k_p\lambda_p}\left(e_p^k - e_p^{k-1}\right) + \frac{T_s}{k_p\lambda_p}e_p^k$$

$$\vartheta^k = \vartheta^{k-1} + \frac{\tau_T}{k_T\lambda_T}\left(e_T^k - e_T^{k-1}\right) + \frac{T_T}{k_T\lambda_T}e_p^k,$$

where e_p^k and e_T^k are the errors in inventory and surrounding temperature, respectively, at instant k. A sampling time $T_s =$ 20 s was used in all the simulation experiments. The parameters and filter constants employed are summarized in Table 2. The inventory y was estimated through the observer (Eq. 48) and the integral form of Eq. 52, with measurements taken at $x = L$. The observer gain was chosen on the basis of a predefined convergence rate, its values being

$$\omega = (5.0, 0.2, 0.1, 0.05, 0.02)^T.$$

Results on temperature evolution as well as movements of the manipulated variables are presented in Figures 4 and 5 for different filter constants. In Figure 4, the estimated inventory y is also represented together with that of the plant, showing very good agreement. Figures 6 represents the evolution of temperature distribution in the product in deviation form with respect to the chosen reference, demonstrating that the uniformity requirement was met in accordance with theoretical results.

Conclusions

In this work, passive control design was demonstrated for a general class of nonlinear distributed systems, representative of relevant operations in chemical and processing technology. Passivity conditions were established on the basis of a reduced number of design parameters associated with the volume of the domain and the rates of transfer and production. These conditions suggested a number of simple control structures that, with very little information about the system, were shown to ensure exponential convergence of the complete field to its prespecified reference. Accurate reconstruction of the field from a limited number of measurements was also linked to the notion of passivity and demonstrated both at theoretical and application levels.

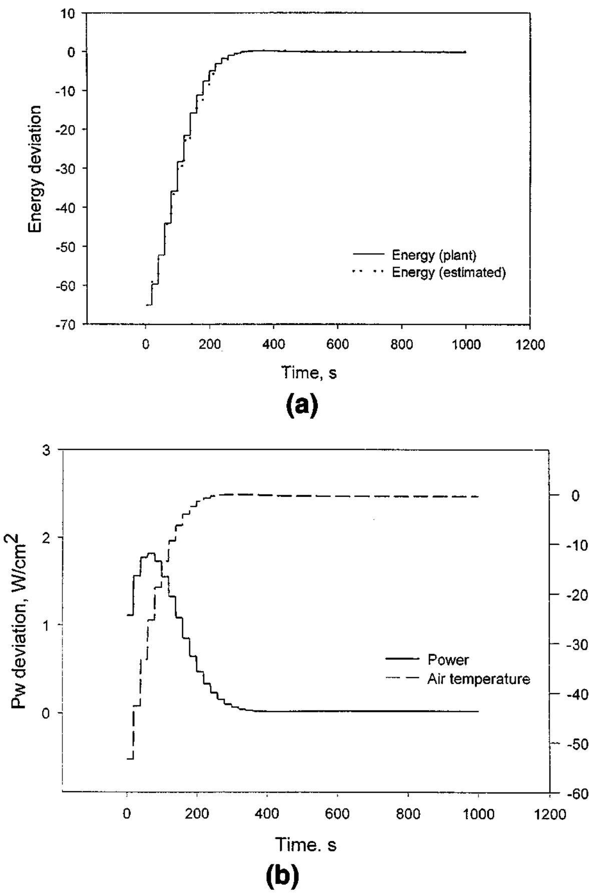


Figure 4. Inventory control for microwave processing $\lambda_p = 20$ s, $\lambda_T = 400$ s.

(a) Evolution of the true and estimated inventory; (b) power and surrounding temperature evolution.

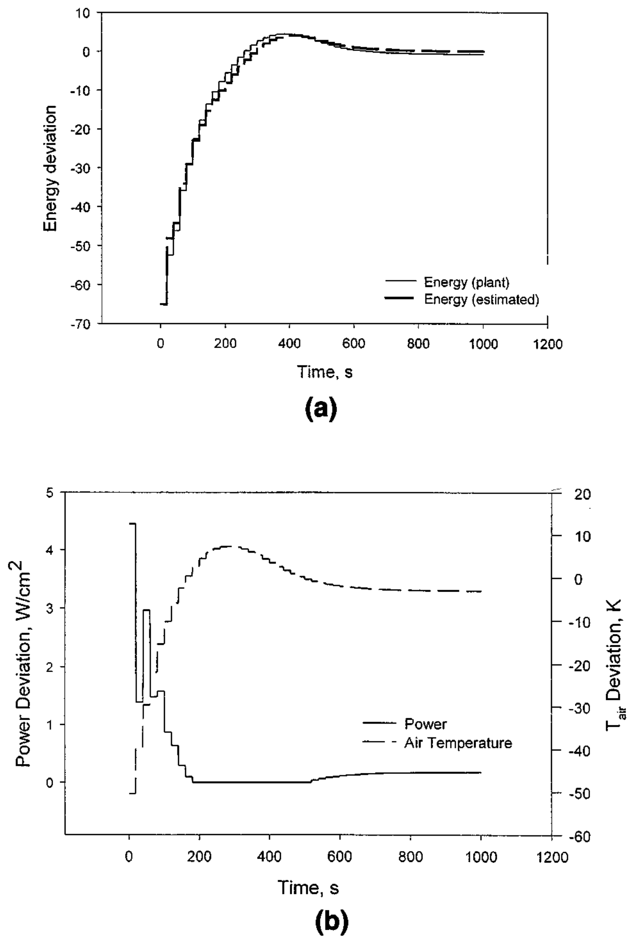


Figure 5. Inventory control for microwave processing $\lambda_p = 80$ s, $\lambda_T = 450$ s.

(a) Evolution of the true and estimated inventory; (b) power and surrounding temperature evolution.

Acknowledgments

The authors acknowledge generous support from the EC Project Fair CT96-1192.

Literature Cited

Alonso, A. A., and B. E. Ydstie, "Process Systems, Passivity and the Second Law of Thermodynamics," *Comput. Chem. Eng.*, **20**, S1119 (1996).

Ayappa, K. G., H. T. Davis, E. A. Davis, and J. Gordon, "Analysis of Microwave Heating of Materials with Temperature Dependent Properties," *AIChE J.*, **37**, 313 (1991).

Balas, M. J., "Trends in Large Scale Structure Control Theory: Fondest Hopes, Wildest Dreams," *IEEE Trans. Automat. Cont.*, **AC-27**, 522 (1982).

Beale, G. O., and M. Li, "Robust Temperature Control for Microwave Heating of Ceramics," *IEEE Trans. Ind. Electron. Contr., Instrum.*, **44**(1), 124 (1997).

Callen, H. B., *Thermodynamics and an Introduction to Thermostatics*, Wiley, New York (1985).

Chen, C. C., and P. C. Chang, "Accelerated Disturbance Damping of an Unknown Distributed System by Nonlinear Feedback," *AIChE J.*, **38**, 1461 (1992).

Christofides, P. D., and P. Daoutidis, "Nonlinear Control of Diffusion-Convection-Reaction Processes," *Comput. Chem. Eng.*, **20**, 1071 (1996).

Christofides, P. D., and P. Daoutidis, "Control of Nonlinear Distributed Parameter Processes: Recent Results and Future Research Directions," *Chemical Process Control IV*, AIChE Symp. Ser., Vol. 93, Amer. Inst. Chem. Eng., New York, p. 302 (1997a).

Christofides, P. D., and P. Daoutidis, "Finite-Dimensional Control of Parabolic PDE Systems Using Approximate Inertial Manifolds," *J. Math. Anal. Appl.*, **216**, 398 (1997b).

Christofides, P. D., "Robust Control of Parabolic PDE Systems," *Chem. Eng. Sci.*, **53**, 2949 (1998).

Courant, R., and D. Hilbert, *Methods of Mathematical Physics*, Wiley, New York (1937).

Cooper, D. J., W. F. Ramirez, and D. E. Clough, "Comparison of Linear Distributed-Parameter Filters to Lumped Approximants," *AIChE J.*, **32**, 186 (1986).

deGroot, S. R., and P. Mazur, *Non-Equilibrium Thermodynamics*, North-Holland, Amsterdam (1962).

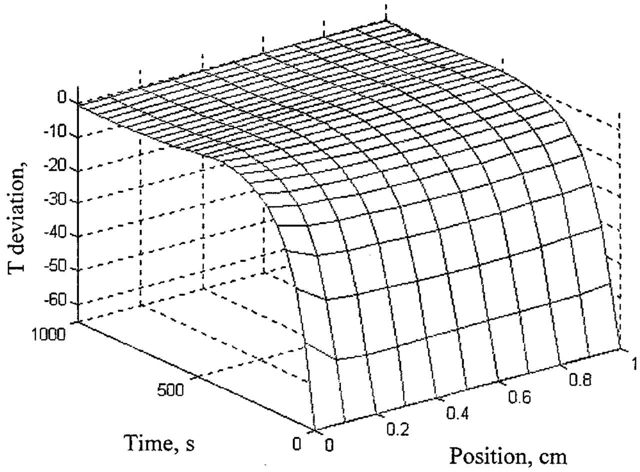
Dennis, J. R., and J. E. Schnabel, *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*, Prentice Hall, Englewood Cliffs, NJ (1983).

Dochain, D., J. P. Babary, and N. Tali-Maamar, "Modelling and Adaptive Control of Nonlinear Distributed Parameter Bioreactors via Orthogonal Collocation," *Automatica*, **28**, 873 (1992).

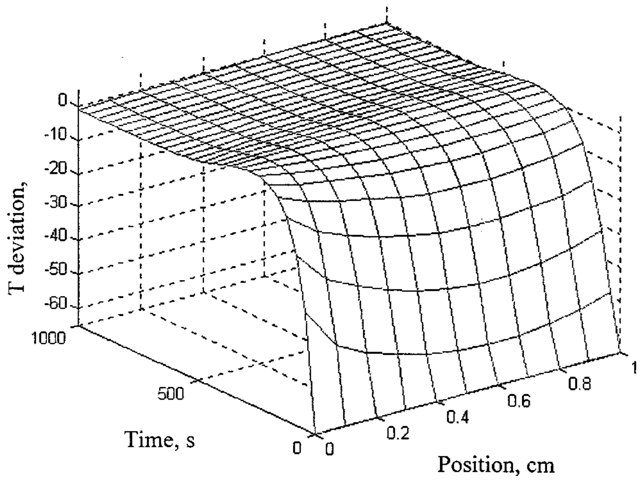
Dochain, D., N. Tali-Maamar, and J. P. Babary, "On Modelling, Monitoring and Control of Fixed Bed Reactors," *Comput. Chem. Eng.*, **21**, 1255 (1997).

Farschman, C. A., K Viswanath, B. E. Ydstie, "Process Systems and Inventory Control," *AIChE J.*, **44**, 1841 (1998).

Fossen, T. I., and J. P. Strand, "Passive Nonlinear Observer Design for Ships Using Lyapunov Methods: Full-Scale Experiments with a Supply Vessel," *Automatica*, **35**, 3 (1999).



(a)



(b)

Figure 6. Temperature deviation distribution for the different filter constants considered.

(a) $\lambda_p = 20$ s, $\lambda_T = 400$ s. (b) $\lambda_p = 80$ s, $\lambda_T = 450$ s.

Fukushima, H., T. Yamanaka, and M. Matsui, "Microwave Heating of Ceramics and Its Application to Joining," *J. Mater. Res.*, **6**, 397 (1990).

Gay, D. H., and W. H. Ray, "Identification and Control of Distributed Parameter Systems by Means of the Singular Value Decomposition," *Chem. Eng. Sci.*, **50**, 1519 (1995).

Hangos, K. M., A. A. Alonso, J. D. Perkins, and B. E. Ydstie, "Thermodynamic Approach to the Structural Stability of Process Plants," *AIChE J.*, **45**, 802 (1999).

Jou, D., J. Casas-Vazquez, and G. Lebon, *Extended Irreversible Thermodynamics*, Springer-Verlag, New York (1993).

Kailath, T., *Linear Systems*, Prentice Hall, Englewood Cliffs, NJ (1980).

Lasiecka, I., "Control of Systems Governed by Partial Differential Equations: A Historical Perspective," *Proc. IEEE Conf. on Decision and Control*, New Orleans, LA, 2792 (1995).

Morari, M., and E. Zafiriou, *Robust Process Control*, Prentice Hall, Englewood Cliffs, NJ (1989).

Ohlsson, T., "The C-Value as a Tool in Optimizing Thermal Processes," *SIK Service Series No.* 786, Gotteborg, Sweden (1988).

Ortega, R., A. Loria, P. J. Nicklasson, and H. Sira-Ramirez, *Passivity-Based Control of Euler-Lagrange Systems*, Springer-Verlag, Berlin (1998).

Ray, W. H.,"Some Recent Applications of Distributed Parameter Systems Theory—A Survey," *Automatica*, **14**, 281 (1978).

Sepulchre, R., M. Jankovic, and P. Kokotovic, *Constructive Nonlinear Control*, Springer-Verlag, London (1997).

Shvartsman, S. Y., and I. G. Kevrekidis, "Nonlinear Model Reduction for Control of Distributed Systems: A Computer Assisted Study," *AIChE J.*, **44**, 1579 (1998).

Sira-Ramirez, H., "General Canonical Form for Feedback Passivity of Nonlinear Systems," *Int. J. Control*, **71**, 891 (1998).

Slotine, J. J., and W. Li, *Applied Nonlinear Control*, Prentice Hall, Englewood Cliffs, NJ (1991).

Smoller, J., *Shock Waves and Reaction-Diffusion Equations*, Springer-Verlag, New York (1983).

Waldraff, W., D. Dochain, S. Bourrel, and A. Magnus, "On the Use of the Observability Measures for Sensor Location in Tubular Reactors," *J. Proc. Control*, **8**, 497 (1998).

Ydstie, B. E., and A. A. Alonso, "Process Systems and Passivity Via the Clausius-Plank Inequality," *Syst. Control. Lett.*, **30**, 253 (1997).

Appendix

The definition and properties of the storage employed are presented next.

Lemma A1. Let $h(x)$ be a concave function with $x \in R^n$. Then, for any pair of vectors x_1 and x_2, we have that

$$h(x_2) \leq h(x_1) + \left(\partial_{x_k} h\right)_{x_1} (x_2 - x_1).$$

Proof. The result follows from concavity of h by constructing a bounding hyperplane (righthand side of the inequality) tangent to h at x_1.

Theorem A1. Let $s(z)$ be strictly concave, $A = \partial_z s$, $b(z;z^*) = A^{T*}(z - z^*) - (s - s^*)$ and $B = \int_{\mathcal{V}} b d\mathcal{V}$, then

1. b is a nonnegative, strictly convex, and reaches its minimum at $z = z^*$;

2. There exists a negative-definite matrix Q such that

$$A - A^* = Q(z - z^*);$$

3. There exists a positive constant β such that

$$B(z, z^*) \leq \beta^{-1} < (A - A^*), (A - A^*) >_{\mathcal{V}}.$$

Proof. That b is nonnegative follows from its definition and Lemma A1 by choosing $h(x) = s(z)$, $x_2 = z$, and $x_1 = z^*$. To show convexity, differentiate b with respect to z, so that

$$\partial_z b = -(A^T - A^{T*}) \quad \partial_{z_i z_j} b = -\partial_z A^T,$$

since s is strictly concave $\partial_{z_i z_j} s = \partial_z A^T < 0$, and therefore $\partial_{z_i z_j} b > 0$ (strictly convex). Moreover $b(z^*;\, z^*) = 0$ so that the function reaches the minimum at $z = z^*$.

To prove result 2, we use Lemma A1 with $h = -b$, $x_2 = z$, and $x_1 = z'$ to obtain the following inequality

$$b(z) \geq b(z') - \left(A^T(z') - A^{T*}\right)(z - z').$$

The inequality is valid for every z and z'. Since for $z = z^*$, $b(z^*) = 0$, it follows that

$$b(z) \leq -\left[A^T(z) - A^T(z^*)\right](z - z^*). \qquad \text{(A1)}$$

Using Newton's theorem of vectorial fields (Dennis and Schnabel, 1983), we have

$$A(z) - A(z^*) = \left[\int_0^1 M(z + \epsilon(z - z^*))\, d\epsilon\right](z - z^*),$$

where $M = \partial_{z_i z_j} s$, $\epsilon \in [0, 1]$ and the integration is carried out elementwise. Since s is concave, M is negative definite and the result follows by defining a matrix Q such that

$$-Q = \int_0^1 M[z + \epsilon(z - z^*)]\, d\epsilon \geq M > 0 \qquad \text{(A2)}$$

$$A(z) - A(z^*) = Q(z - z^*).$$

Result 3 derives from result 2 and inequality A1 by choosing β' as the smallest eigenvalue of matrix M and defining $\beta^{-1} = \sup_{\mathcal{V}} (\beta'^{-1})$ so that

$$b \leq (A^T - A^{*T}) Q^{-1} (A - A^*) \leq \beta'^{-1} \bar{A}^T \bar{A}$$

and the inequality follows by integration on $\mathcal{V}$.

Manuscript received June 7, 1999, and revision received Feb. 21, 2000.